Abstract.

Despite great progress in the study of critical percolation on Z^d for d large, properties of critical clusters in high-dimensional fractional spaces and boxes remain poorly understood, unlike the situation in two dimensions. Closely related models such as critical branching random walk give natural conjectures for the value of the relevant high-dimensional critical exponents; see in particular the conjecture by Kozma-Nachmias that the probability that 0 and (n, n, n, . . .) are connected within $[-n, n]^d$ scales as n^{-2-2d}.

In this paper, we study the properties of critical clusters in high-dimensional half-spaces and boxes. In half-spaces, we show that the probability of an open connection (“arm”) from 0 to the boundary of a side length n box scales as n^{-3}. We also find the scaling of the half-space two-point function (the probability of an open connection between two vertices) and the tail of the cluster size distribution. In boxes, we obtain the scaling of the two-point function between vertices which are any macroscopic distance away from the boundary. Our argument involves a new application of the “mass transport" principle which we expect will be useful to obtain quantitative estimates for a range of other problems.